

Torsion Analysis for Cold-Formed Steel Members Using Flexural Analogies

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Abstract

The design of cold-formed steel members must consider the impact of torsional loads due to transverse load eccentricity. Open cross-sections are particularly susceptible to significant twisting and high warping stresses. Design requirements for combined bending and torsion were introduced in the American Iron and Steel Institute Specification in 2007, and more recently in the Australian/New Zealand Standard 4600:2018. These provisions require an understanding of the distribution of internal forces and stresses due to torsional warping, which is not commonly taught in engineering curriculums. Furthermore, most structural analysis programs do not properly consider torsional warping stiffness and response. The purpose of this paper is to educate the structural engineer on torsion analysis using analogies to familiar flexural relationships. Useful formulas are provided for determining torsional properties and stresses.

1. Introduction

Cold-formed steel members of open cross-section are often susceptible to twisting and torsional stresses. The shear center for many shapes is outside the envelope of the cross-section so it can be difficult to apply transverse loads without producing torsional effects. Open thin-walled members also have inherently low torsional stiffness, thus even small magnitudes of torsion can result in a significant amount of twisting.

Design strategies can be employed to reduce or eliminate torsion. Locating beams in line with transverse loads is often the most obvious approach. Remaining load eccentricity from the beam shear center can be effectively reduced by transferring loads through components that restrain twisting, thereby causing the loads to act through the shear center. For cases where beams cannot be aligned with transverse loads, supplemental perpendicular members could be designed to carry the loads in flexure, rather than subjecting the supporting beam to torsion.

If torsional loads cannot be avoided, the member must be designed with consideration for the resulting twist and torsional stresses. Large torsional loads are best handled using closed cross-sections with high torsional stiffness and minimal warping. Applications using open cross-sections may require additional bracing to restrain twist at spacings close enough to reduce the amount of twist and torsional stresses to acceptable levels.

Current editions of design specifications AISI S100 [1] and AS/NZS 4600 [2] have provisions to account for stresses produced by torsional loads. These provisions consider the effect of combined longitudinal stresses resulting from flexure and torsional warping. Future provisions may address combined shear stresses from flexure and torsion, and may consider the impact of combined longitudinal and shear stresses from all types of loading.

Understanding the distribution of internal torsional forces is essential to performing the required member design calculations. In a flexural analysis, the distribution of moments and shear forces along a beam are independent of the selected member size. But in a torsion analysis, the distribution of internal forces can vary with certain properties of the cross-section. This paper provides an overview of torsion analysis for cold-formed steel members. It is intended to educate the engineer in the characteristics of torsional internal forces, how they are distributed, and ways to simplify these concepts.

2. General Torsion Theory

The development of the elastic torsional response of general prismatic members can be found in the works of Timoshenko [3] and Vlasov [4], which remain the basis for torsional design methods as presented in numerous texts, including Heins [5], Seaburg and Carter [6], and SCI P385 [7]. In the classical theory, the relationship between applied torsion and angle of twist is given by Equation 1:

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$$EC_w \phi'''' - GJ \phi'' = m_t(z) \quad (1)$$

where E is the modulus of elasticity, C_w is the warping torsion constant, G is the shear modulus, J is the St. Venant torsion constant, ϕ is the shape function for angle of twist, and $m_t(z)$ is the applied torsional moment intensity as a function of the longitudinal z axis of the member. The internal torque T is obtained by integrating the applied torsional moment, as given by Equation 2:

$$T(z) = EC_w \phi''' - GJ \phi' = T_w + T_{sv} \quad (2)$$

This internal torque is resisted by two types of shear corresponding to the two terms in Equation 2. The first term is the warping torsion component T_w , and the second term is the St. Venant (pure) torsion component T_{sv} . Some cross-sections as shown in Figure 1 are dominated by pure torsion where EC_w/L^2 is much less than GJ , and others as shown in Figure 2 are often dominated by warping torsion where EC_w/L^2 is much greater than GJ , particularly for thin material.



Figure 1: Sections dominated by pure torsion: $EC_w/L^2 \ll GJ$

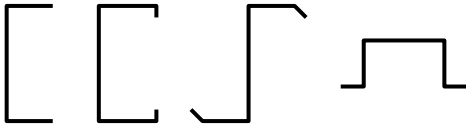


Figure 2: Sections dominated by warping torsion: $EC_w/L^2 \gg GJ$

For cross-sections where torsion is resisted by both pure torsion and warping torsion, the solution to Equation 1 is more complex than beam flexure, requiring the use of hyperbolic sine and cosine functions. This solution is discussed in Section 5. It is helpful to first understand the two torsional mechanisms individually. These are presented in Section 3 for pure torsion and Section 4 for warping torsion.

3. Pure Torsion

A member undergoes pure torsion if the plane of each cross-section remains plane as the member twists (see Figure 3). This type of torsion occurs only with closed or solid circular cross-sections. However, pure torsion is predominant for other closed cross-sections, and for cross-sections with straight elements passing through the shear center, where the warping constant C_w is relatively small. These are the cases shown in Figure 1.

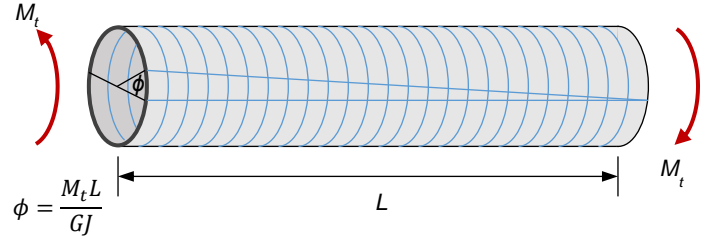


Figure 3: Pure torsion – plane sections remain plane

For the loading in Figure 3 where torsional moments M_t are applied to the ends of the member, the internal torque is constant throughout the length of the member, and the total angle of twist is $M_t L / GJ$.

Pure torsion can be evaluated using an elastic membrane analogy, which was first introduced by Prandtl [8]. In this analogy a membrane over the cross-sectional area displaces due to an internal pressure. The volume under the displaced membrane is proportional to the internal torque, and the slope of the membrane is proportional to the shear stress.

For common shapes, the torsion constant and shear stresses can be closely approximated with simple formulas. The development of these can be found in several texts such as Muvdi and McNabb [9]. For closed cross-sections with uniform thickness t , the torsion constant is calculated using Equation 3, where A_m is the area enclosed by the midline of the thickness, and S_m is the length of the midline perimeter, as illustrated in Figure 4.

$$J = \frac{4A_m^2 t}{S_m} \quad (\text{closed sections}) \quad (3)$$

For the special case of a cylindrical tube, $A_m = \pi r_m^2$ and $S_m = 2\pi r_m$, giving the torsion constant as:

$$J = 2\pi r_m^3 t \quad (\text{cylindrical tubes}) \quad (4)$$

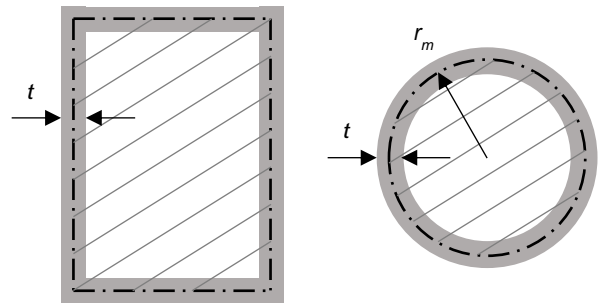


Figure 4: St. Venant torsion constant for closed sections

For open sections, the torsion constant is calculated using Equation 5. For uniform thickness, this simplifies to $J = \frac{1}{3}At^2$, where A is the cross-sectional area.

$$J = \frac{1}{3} \int t^3 ds = \frac{1}{3} \sum bt^3 \quad (\text{open sections}) \quad (5)$$

The flow of shear stress caused by pure torsion is illustrated in Figure 5. Closed sections have a nearly constant stress across the thickness, and open sections have linearly varying stresses across the thickness.

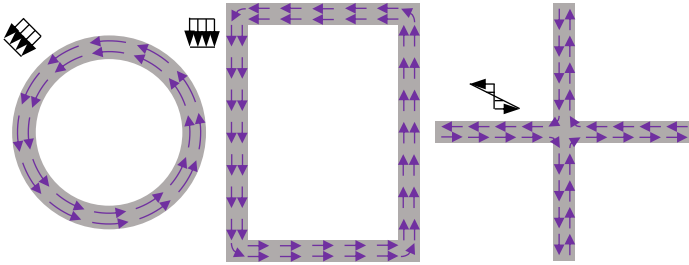


Figure 5: St. Venant shear stresses

For closed sections with uniform thickness, shear stress is calculated using Equation 6, which simplifies to Equation 7 for cylindrical tubes. For open sections, the maximum shear stress at the material surface is calculated using Equation 8.

$$\tau_{sv} = \frac{T_{sv}}{2A_m t} \quad (\text{closed sections}) \quad (6)$$

$$\tau_{sv} = \frac{T_{sv} r_m}{J} \quad (\text{cylindrical tubes}) \quad (7)$$

$$\tau_{sv} = \frac{T_{sv} t}{J} \quad (\text{open sections}) \quad (8)$$

Determining the internal torque T_{sv} at any point along the member requires an understanding of the torsion distribution. For pure torsion, the first term in Equation 1 is omitted, and the second derivative of the displacement function is proportional to the torsion intensity m_t . Integration of the torsion intensity gives the internal torque T_{sv} , which is proportional to the first derivative of the displacement function. Finally, integration of internal torque divided by the torsional stiffness (T_{sv}/GJ) and changing sign, gives the displacement twist angle ϕ .

These relationships are illustrated in Figure 6. Structural engineers learn the relationships between load, shear, and moment diagrams for beam flexure, and these concepts can be applied to pure torsion relationships. The area under the torsion intensity curve between two points along the member is equal to the change in internal torque between those two points, and the slope of the internal torque curve at any point is equal to the magnitude of torsion intensity.

The area under the internal torque curve between two points is equal to the change in $-GJ\phi$ between those two points, and the slope of the twist angle curve at any point is equal to $-T_{sv}/GJ$.

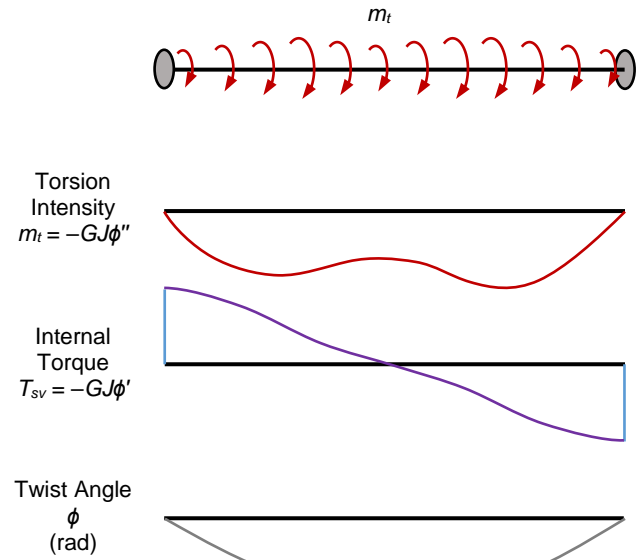


Figure 6: Pure torsion distribution

For a simple beam with uniform torsion, the diagrams for internal torque and twist angle are shown in Figure 7. The expressions for maximum torque and twist angle should look familiar to the structural engineer, as they mirror the flexural diagrams for shear and moment for a simple beam with uniform load.

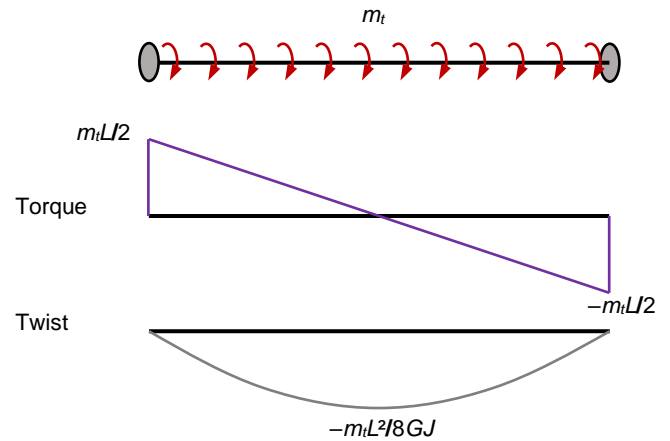


Figure 7: Pure torsion distribution

This analogy between beam flexure and pure torsion can be extended to multiple span applications and different loading patterns (see Appendix A for more examples). It is then convenient to use common beam tables and formulas to determine internal torque and twist angle in pure torsion applications, as summarized in Table 1.

Table 1: Beam table usage for pure torsion

Flexure		Pure Torsion	
Pinned Support		Twisting Restraint	
Concentrated Load	P	Concentrated Torsion	M_t
Uniform Load	w	Uniform Torsion	m_t
Reaction Force	R	Reaction Torque	R_t
Internal Shear	V	Internal Torque	T_{sv}
Internal Moment	M	Proportional to Twist	$-GJ\phi$

4. Warping Torsion

A member undergoes warping torsion if the plane of each cross-section does not remain plane as the member twists, but warps as shown in Figure 8. Open cross-sections generally experience warping torsion, and with thinner material (smaller J) warping torsion is predominant. In this section, warping torsion will be evaluated with the assumption that St. Venant torsion is negligible.

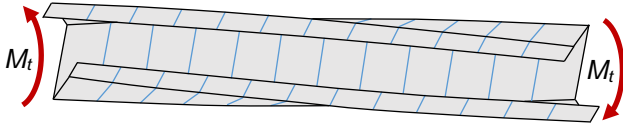


Figure 8: Warping torsion – plane sections warp out of plane

For the loading in Figure 8 where torsional moments are applied to the ends of the member and longitudinal warping is unrestrained, warping occurs uniformly throughout the length of the member and no warping stresses develop. For cases where warping is restrained or torsional loads are applied between twisting restraints, the amount of warping varies along the length of the member, which produces longitudinal and shear stresses in the member. In order to design a member for torsion, it is essential to understand the characteristics of these stresses and their combined effects.

Flexural members experience longitudinal stresses and shear stresses which vary along the length of the member and throughout the cross-section. Structural engineers separate these stresses into the cross-sectional stress resultants of moments and shear forces to isolate failure modes. For a simple beam, the maximum moment occurs within the span where the shear force is zero, and the maximum shear force occurs at the ends of the span where the moment is zero. For other end conditions and multi-span beams, the maximum moment and maximum shear can occur at the same location. The design strength for this combined stress state is determined with an interaction equation using the stress resultants.

Similarly, members undergoing warping torsion experience longitudinal stresses and shear stresses which vary along the length of the member and throughout the cross-section.

Maximum longitudinal and shear stresses typically occur at different locations. In order to evaluate the combined effects of flexure and warping torsion, it is necessary to utilize the cross-sectional stress resultants from warping torsion which separate longitudinal stresses from shear stresses.

As with pure torsion, certain properties of the cross-section are required to determine warping torsion stiffness and stresses. These properties are the warping constant (C_w), shear center coordinates (x_o , y_o) relative to the centroid, sectorial area from the centroid (w_c), sectorial area from the shear center (w_o), normalized unit warping (w_n), and warping statical moment (S_w). The details of these integrations over the cross-section are available in various texts such as Yu [10].

$$C_w = \int_0^l w_n^2 t \, ds \quad (9)$$

$$x_o = \frac{1}{I_x} \int_0^l w_c y t \, ds \quad (10)$$

$$y_o = \frac{1}{I_y} \int_0^l w_c x t \, ds \quad (11)$$

$$w_c = \int_0^s R_c \, ds \quad (12)$$

$$w_o = \int_0^s R_o \, ds \quad (13)$$

$$w_n = w_o - \frac{1}{A} \int_0^l w_c t \, ds = w_o - \bar{w}_o \quad (14)$$

$$S_w = \int_0^s w_n t \, ds \quad (15)$$

The shear center is the point in the cross-section at which a transverse load will not induce any torsion. The internal torque about the centroid produced by flexural shear stresses is counteracted by the torque from the transverse load applied at the shear center.

The properties w_c , w_o , w_n , and S_w are not for the entire cross-section, but for a specific point in the cross-section at distance s from the starting free edge of an open cross-section. The maximum values of w_n and S_w are sometimes used in design, and may be found in published tables of cross-section properties.

The flow of cross-sectional shear stress caused by warping torsion is illustrated in Figure 9. The shear stress is essentially constant across the thickness, but varies along the length of the elements. The magnitude of the warping shear stress is calculated using Equation 16, where T_w is the internal warping torque and is the resultant torque of the shear stresses over the cross-sectional area.

$$\tau_w = \frac{T_w S_w}{C_w t} \quad (16)$$

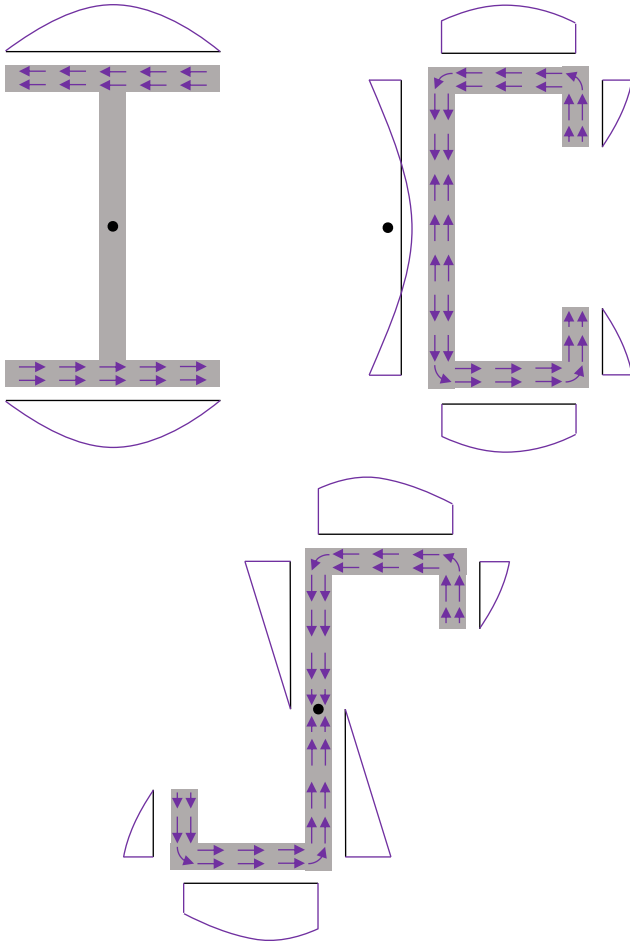


Figure 9: Warping shear stresses

The warping shear stress is proportional to the warping statical moment S_w . Therefore, the stress magnitudes shown in Figure 9 reflect the relative magnitudes of S_w throughout the cross-section.

The longitudinal (normal) stresses which develop in warping torsion are illustrated in Figure 10. The resultant of these stresses is called the warping bimoment, designated as B . This term is less known than the familiar bending moment, but they are both longitudinal stress resultants which are useful in strength determination. The bimoment resultant is the summation of longitudinal stress times the normalized unit warping (w_n) over the area of the cross-section. The stress distribution has no net axial force and no net bending moment.

For the special case of a doubly-symmetric I section as shown in Figure 10, the flanges undergo minor axis bending in opposite directions and produce a moment couple equivalent to the bimoment ($B=M \cdot d$), hence the name. For the general case, a single warping neutral axis does not exist and an equivalent moment couple may not be

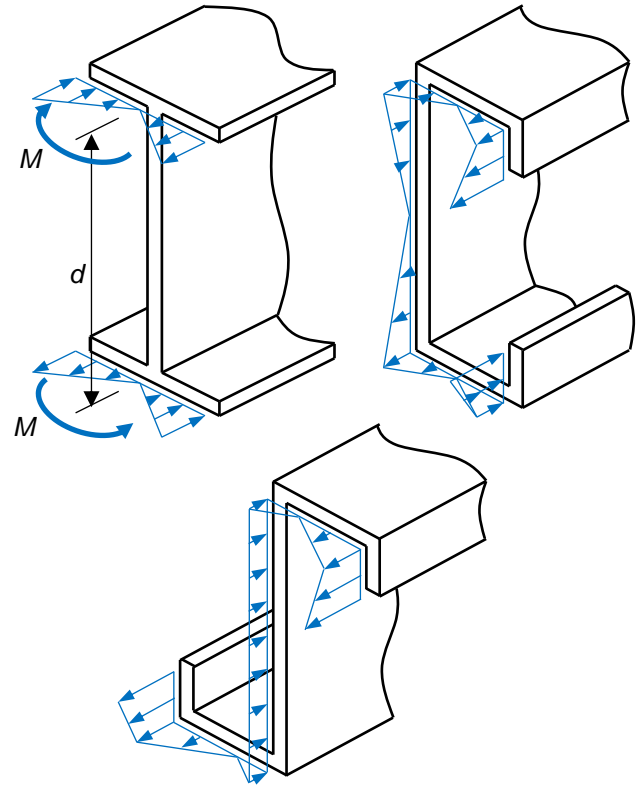


Figure 10: Longitudinal (normal) warping stresses

apparent. But the application of an external moment couple will produce the bimoment stress distribution.

The magnitude of the longitudinal stress is calculated using Equation 17. In this expression, the stress is proportional to the normalized unit warping w_n . Therefore, the stress magnitudes shown in Figure 10 reflect the relative magnitudes of w_n throughout the cross-section.

$$\sigma_w = \frac{B w_n}{C_w} \quad (17)$$

Determining the internal warping torque T_w and bimoment B at any point along the member requires an understanding of the torsion distribution. For warping torsion alone, the second term in Equation 1 is omitted, and the fourth derivative of the displacement function is proportional to the torsion intensity m_t . Integration of the torsion intensity gives the internal torque T_w , which is proportional to the third derivative of the displacement function. Integration of the internal torque gives the bimoment B , which is proportional to the second derivative of the displacement function. Longitudinal warping displacement is the first derivative of the displacement function times the normalized unit warping w_n .

These relationships are illustrated in Figure 11. Structural engineers learn the relationships between load, shear, moment, rotation, and deflection diagrams for beam flexure, and these concepts can be directly applied to warping torsion relationships. The area under the torsion intensity curve between two points along the member is equal to the change in warping torque between those two points, and the slope of the warping torque curve at any point is equal to the magnitude of torsion intensity. The area under the warping torque curve between two points is equal to the change in bimoment between those two points, and the slope of the bimoment curve at any point is equal to the warping torque.

Table 2: Flexure – warping torsion analogy

Flexure	Pure Torsion
Differential Equation $El_x v'''' = w(z)$	Differential Equation $EC_w \phi'''' = m_t(z)$
Moment of Inertia $I_x = \int y^2 dA$	Warping Constant $C_w = \int w_n^2 dA$
First Moment of Area $Q = \int y dA$	First Sectorial Moment $S_w = \int w_n dA$
Section Coordinate y	Sectorial Coordinate w_n
Deflection v	Twist ϕ
Load Intensity w	Torsion Intensity m_t
Bending Moment $M_x = El_x v''$	Warping Bimoment $B = EC_w \phi''$
Bending Stress $\sigma_b = M_x y / I_x$	Warping Stress $\sigma_w = B w_n / C_w$
Stress Resultant $M_x = \int \sigma_b y dA$	Stress Resultant $B = \int \sigma_w w_n dA$
Shear Force $V = El_x v'''$	Warping Torque $T_w = EC_w \phi'''$
Shear Stress $\tau = VQ / I_x t$	Warping Shear Stress $\tau_w = T_w S_w / C_w t$

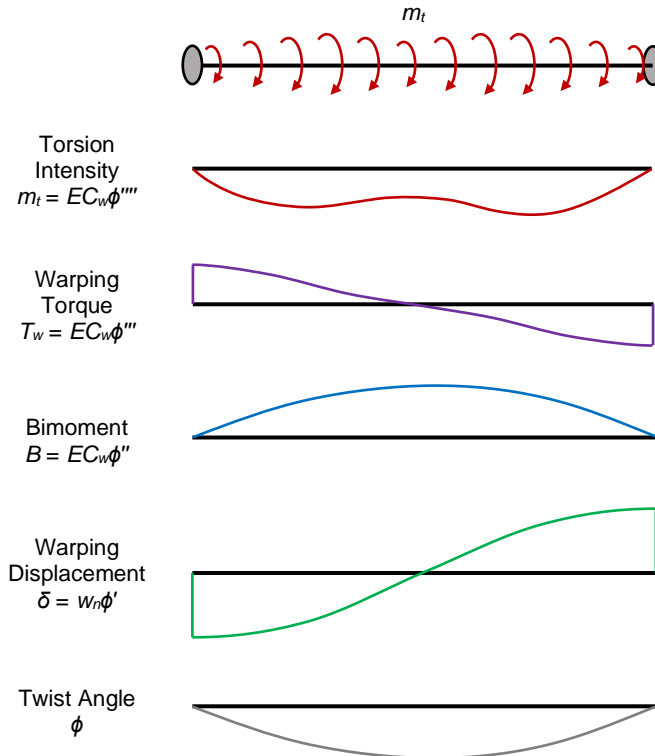


Figure 11: Warping torsion distribution

The analogy between flexure and warping torsion goes beyond the similarities observed in these distribution diagrams. Table 2 provides an extended side-by-side comparison.

Some torsion references have negative signs in some of these warping torsion expressions, which is simply a matter of the sign conventions used. For cross-sections which are not doubly-symmetric, it is important to understand the sign conventions when evaluating combined stresses from flexure and torsion. Figure 12 provides the sign conventions used in this paper. Positive longitudinal stresses for flexure (σ_b) and warping torsion (σ_w) are in compression.

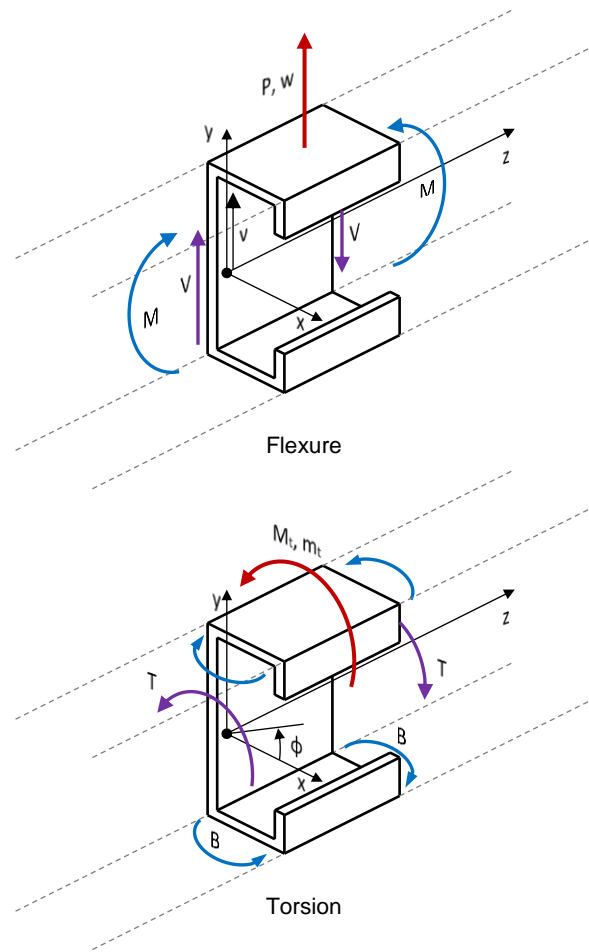


Figure 12: Sign conventions

For a simple beam with uniform torsion, the diagrams for internal torque, bimoment, and twist angle are shown in Figure 13. The expressions for maximum torque, bimoment, and twist angle should look familiar to the structural engineer, as they mirror the flexural diagrams for shear, moment, and deflection for a simple beam with uniform load.

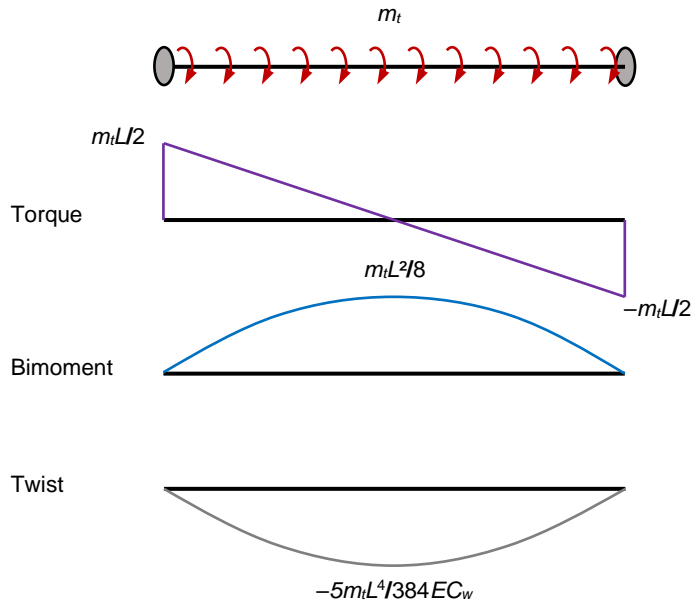


Figure 13: Sign conventions

This analogy between beam flexure and warping torsion can be extended to other support conditions, multiple span applications, and different loading patterns (see Appendix B for more examples). Pinned supports for flexure correspond to twisting restraints for warping torsion. Fixed rotations for flexure correspond to warping restraints for torsion. It is then convenient to use common beam tables and formulas to determine torque, bimoment, and twist angle in warping torsion applications, as summarized in Table 3.

Table 3: Beam table usage for warping torsion

Flexure		Pure Torsion	
Pinned Support		Twisting Restraint	
Fixed Rotation		Warping Restraint	
Concentrated Load	P	Concentrated Torsion	M_t
Uniform Load	w	Uniform Torsion	m_t
Reaction Force	R	Reaction Torque	R_t
Internal Shear	V	Internal Torque	T_{sv}
Internal Moment	M	Internal Bimoment	B
Moment of Inertia	I_x	Warping Constant	C_w
Deflection	Δ	Twist Angle	ϕ

5. Combined Warping and St. Venant Torsion

Sections that do not behave predominantly as either pure torsion or warping torsion exhibit both types of torsion simultaneously. This is common for open sections having thicker material, where the St. Venant torsion constant is no longer negligible. The degree to which torsional forces are resisted by pure torsion and warping torsion is a function of their relative torsional stiffnesses. The stiffness for pure torsion is GJ , and the stiffness for warping torsion is EC_w/L^2 .

It is common to utilize a property of the cross-section (a) that captures the relative torsional stiffness potential, which is given by Equation 18. This property has a dimension of length and is sometimes provided in published tables of cross-section properties. Cross-sections with smaller values exhibit more pure torsion, whereas those with larger values exhibit more warping torsion.

$$a = \sqrt{\frac{EC_w}{GJ}} \quad (18)$$

This property alone does not fully capture the relative torsional stiffness because the warping stiffness involves the member length. Therefore, the ratio L/a is used to truly capture the relative contributions of pure torsion and warping torsion. Smaller values of L/a have more warping torsion, larger values have more pure torsion. This is demonstrated in Figure 14 for a simply supported beam with uniform torsion.

For L/a less than 0.1, pure torsion is negligible and the torsion response can be treated as warping only. As L/a increases, the total internal torque does not change, but warping torque decreases as St. Venant torque increases. The bimoment and twist angle decrease from the warping only magnitudes. For L/a greater than 20, warping torsion is negligible and the torsion response can be treated as pure torsion only.

The ratio of maximum bimoment (B) to that for warping only (B_{wo}) is plotted vs. L/a in Figure 15 for a simply supported span. Two cases are shown, one for uniformly distributed torsion and one for concentrated torsion applied at mid-span. For a wide variety of standard C-section studs defined by SFIA [12] with a reasonable torsion span 10 times the web depth (h), L/a is small and warping torsion dominates. For a relatively long torsion span of $20h$, there is only a moderate reduction in bimoment.

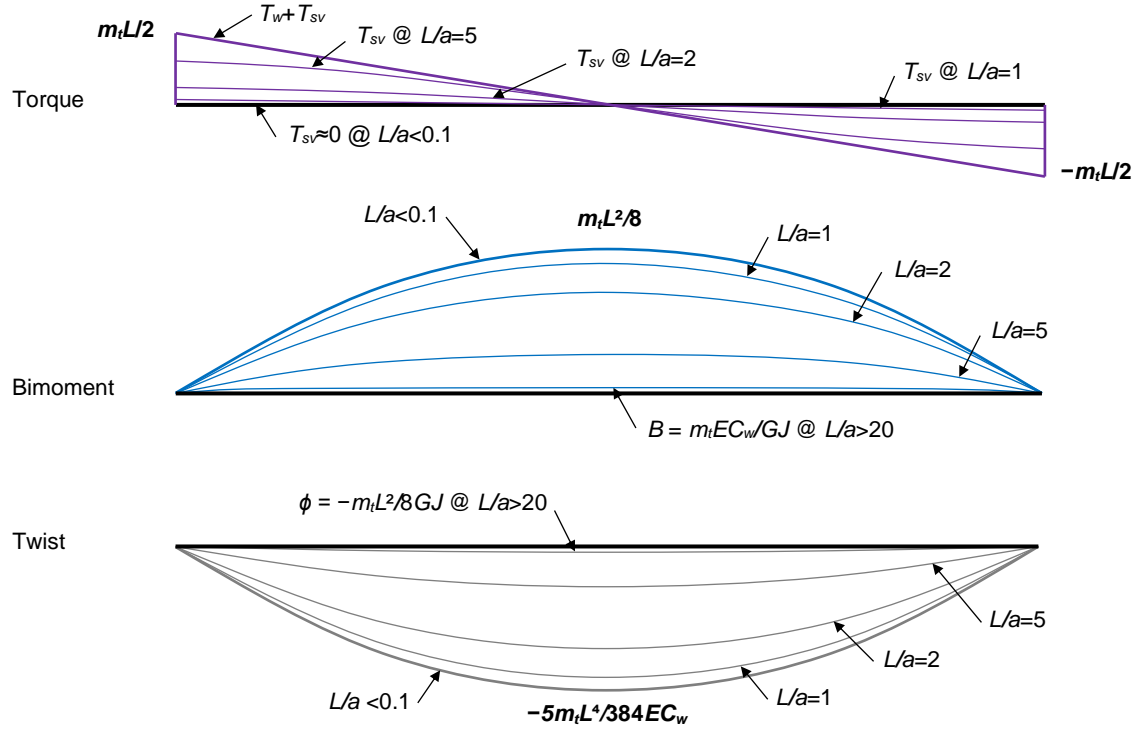


Figure 14: Influence of L/a on warping torsion

Quantifying the actual contributions of pure torsion and warping torsion is more complex than the flexural analogies for pure and warping torsion acting alone. For a trapezoidal loading $m_t(z) = m_t + \Delta m_t z/L$, the general solution to Equation 1 is given as Equation 19, with successive derivatives provided in Equations 20 to 23.

$$\phi = C_1 + C_2 z + C_3 \sinh(z/a) + C_4 \cosh(z/a) - m_t z^2 / 2GJ - \Delta m_t z^3 / 6GJL \quad (19)$$

$$\phi' = C_2 + (C_3/a) \cosh(z/a) + (C_4/a) \sinh(z/a) - m_t z / GJ - \Delta m_t z^2 / 2GJL \quad (20)$$

$$\phi'' = (C_3/a^2) \sinh(z/a) + (C_4/a^2) \cosh(z/a) - m_t / GJ - \Delta m_t z / GJL \quad (21)$$

$$\phi''' = (C_3/a^3) \cosh(z/a) + (C_4/a^3) \sinh(z/a) - \Delta m_t / GJL \quad (22)$$

$$\phi'''' = (C_3/a^4) \sinh(z/a) + (C_4/a^4) \cosh(z/a) \quad (23)$$

Two boundary conditions at each end of the member provide four equations which are necessary to determine the four constants C_1 , C_2 , C_3 , and C_4 . Table 4 summarizes the possible cases, where z represents the longitudinal positions at the ends of the member (0 or L).

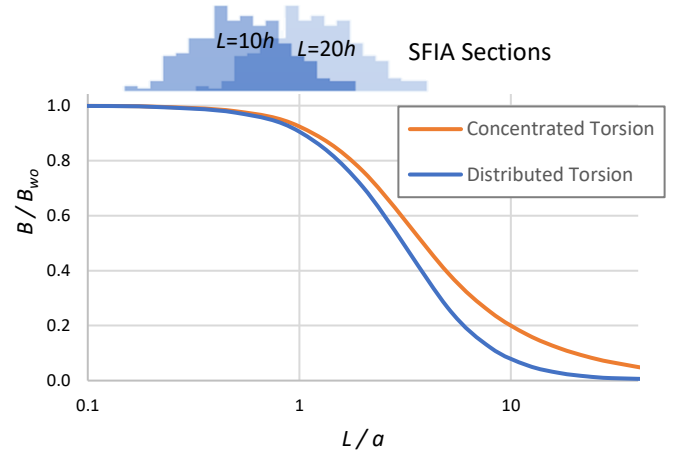


Figure 15: Influence of L/a on bimoment

Table 4: Boundary condition equations

Boundary Condition	Equation
Twisting Restrained	$\phi_z = 0$
Twisting Free	$T = EC_w \phi_z''' - GJ \phi_z' = 0$
Warping Restrained	$\phi_z' = 0$
Warping Free	$B = EC_w \phi_z'' = 0$

For member analyses involving multiple torsion segments, each segment requires four equations and continuity must be enforced from one segment to the next. If twisting is not restrained at the adjoining point, ϕ and T must be equated between the two segments. If warping is not restrained at the adjoining point, ϕ' and B must be equated between the two segments.

Due to the complexity of the complete solution, approximate methods can be useful. For members with L/a less than 0.1 or greater than 20, torque and bimoment can be determined using the flexural analogies for warping torsion alone or pure torsion alone, respectively. For members with moderately small L/a (less than 2), the bimoment determined for warping torsion alone could be conservatively used for design.

For intermediate values of L/a , the pure and warping torsion responses could be approximated by proportioning the load they each carry based on their relative stiffness, such as springs in parallel. For example, consider a simple span with uniformly distributed torsion $m_t = 0.024$ kN-mm/mm, $L = 3000$ mm, $E = 203$ kN/mm², $C_w = 900 \times 10^6$ mm⁶, $G = 78$ kN/mm², $J = 1500$ mm⁴, $a = 1250$ mm, $L/a = 2.4$.

$$\phi_{sv0} = -m_t L^2 / 8GJ = -0.231 \text{ rad at midspan}$$

$$\phi_{wo} = -5m_t L^4 / 384EC_w = -0.139 \text{ rad at midspan}$$

$$1/\phi = 1/\phi_{sv0} + 1/\phi_{wo}, \quad \phi = -0.0868 \text{ rad at midspan}$$

$$m_{tsv} = \left(\frac{\phi}{\phi_{sv0}} \right) m_t = 0.009 \text{ kN-mm/mm}$$

$$m_{tw} = \left(\frac{\phi}{\phi_{wo}} \right) m_t = 0.015 \text{ kN-mm/mm}$$

$$T_{sv} = m_{tsv} L / 2 = 13.5 \text{ kN-mm at support}$$

$$T_w = m_{tw} L / 2 = 22.5 \text{ kN-mm at support}$$

$$B = m_{tw} L^2 / 8 = 16,900 \text{ kN-mm}^2 \text{ at midspan}$$

These results are close to the correct values for twist and bimoment ($\phi = -0.0872$ rad, $B = 16,800$ kN-mm²), accurate for total torque ($T = 36.0$ kN-mm), and approximate for torque components ($T_{sv} = 11.0$ kN-mm, $T_w = 25.0$ kN-mm).

For numerical methods, it is convenient and efficient to approximate Equation 19 as a cubic equation, where the third and fourth terms are $C_3 z^2$ and $C_4 z^3$, respectively (see [13]). This can provide a reasonably close solution for displacement, with approximate parabolic St. Venant torque. However, the bimoment is a linear expression and the warping torque is constant. The amount of error resulting from this method can be reduced by discretizing the member into multiple segments.

6. Analysis Software

Shell finite element analysis with sufficient number of elements can properly simulate torsion behavior including warping. Torsional stiffness, displacements, and stresses can be predicted, but the internal forces required for design (B , T_w , T_{sv}) are not available. Structural analysis applications using beam elements commonly handle pure torsion, but many do not consider warping torsion, or they use assumptions that are not sufficient for cold-formed steel design. Most do not accommodate torsional loads applied along beam elements.

MASTAN2 [14] is a robust structural analysis application that addresses warping torsion for cold-formed steel members. It utilizes an additional degree of freedom at each end of a beam element for warping. The warping displacement function uses a cubic equation as discussed in Section 5. MASTAN2 [14] also offers rigorous second-order analysis. The RF-/STEEL [15] application has similar capabilities.

The CFS® [16] software provides first-order analysis and design for cold-formed steel continuous beams and beam-columns. Pure and warping torsion are rigorously handled using the complete solution as described in Section 5. Torsional loads are automatically determined from the applied transverse loads (concentrated or distributed) and their eccentricities relative to the member shear centers. The AISI [1] provisions for combined bending and torsion are incorporated in member design checks.

7. Conclusions

The fundamentals of cold-formed steel torsion behavior were presented. Formulas for the determination of torsional shear and longitudinal stresses require knowledge of torsion distribution along the member. The analogies given between flexural response and torsion response provide useful tools for the structural engineer to better understand the distribution of both pure torsion and warping torsion. The torsion capabilities and limitations of structural analysis software should be understood and used with care when considering torsion response in the solution.

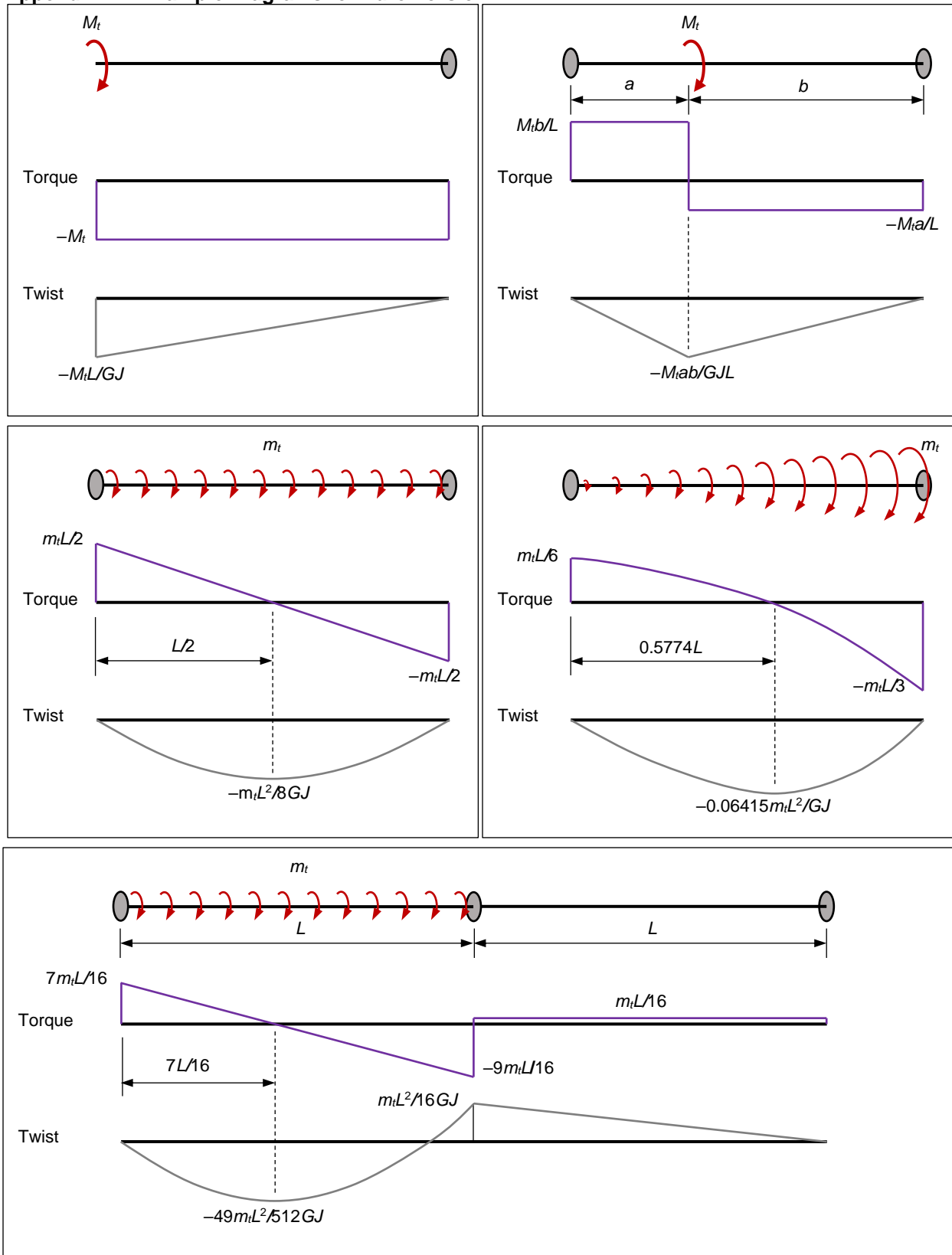
Notation

A	Cross-sectional area (L^2)
A_m	Area enclosed by midline of thickness (L^2)
B	Internal bimoment (FL^2)
B_{wo}	Internal bimoment for warping response only (FL^2)
C_w	Warping torsion constant (L^6)
E	Modulus of elasticity (F/L^2)
G	Shear modulus (F/L^2)
I_x	Moment of inertia about x axis (L^4)
J	St. Venant torsion constant (L^4)
L	Length of torsion span (L)
M	Internal bending moment (FL)
M_t	Concentrated torsional moment (FL)
m_t	Distributed torsional moment intensity (FL/L)
m_{tsv}	Distributed torsion for St. Venant response (FL/L)
m_{tw}	Distributed torsion for warping response (FL/L)
P	Concentrated transverse load (F)
Q	First moment of cross-sectional area (L^3)
R	Reaction force due to transverse loads (F)
R_t	Reaction torque due to torsion loads (FL)
r_m	Radius of cylindrical tube at midline of thickness (L)
S_m	Length of perimeter at midline of thickness (L)
S_w	Warping statical moment (L^4)
s	Distance along midline of thickness from starting free edge of an open cross-section (L)
T	Internal total torque (FL)
T_w	Internal warping torque (FL)
T_{sv}	Internal St. Venant (pure) torque (FL)
t	Material thickness (L)
V	Internal shear force due to flexure (F)
w	Distributed transverse load intensity (F/L)
w_c	Sectorial area from centroid (L^2)
w_n	Normalized unit warping (L^2)
w_o	Sectorial area from shear center (L^2)
x_o, y_o	Shear center coordinates relative to centroid (L)
x, y	Cross-section coordinates relative to centroid (L)
z	Coordinate along longitudinal axis of member (L)
ϕ	Twist angle (rad)
ϕ_{svo}	Twist angle for St. Venant response only (rad)
ϕ_{wo}	Twist angle for warping response only (rad)
v	Transverse deflection due to flexure (L)
σ_w	Longitudinal (normal) warping stress (F/L^2)
σ_b	Longitudinal (normal) bending stress (F/L^2)
τ	Shear stress (F/L^2)
τ_{sv}	Shear stress due to St. Venant torsion (F/L^2)
τ_w	Shear stress due to warping torsion (F/L^2)

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Appendix A – Example Diagrams for Pure Torsion



Appendix B – Example Diagrams for Warping Torsion

